Science & Global Security, 1990, Volume 1, pp.225–302
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Printed in the United States of America

Appendix B

EMISSION AND ABSORPTION OF RADIATION

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A full understanding of fissile-material detection requires a solid knowledge of nuclear physics, especially the emission and absorption of radiation. The purpose of this appendix is therefore to give a brief explanation of these processes so that the reader can better understand the assumptions leading to the results in the main text.

THE NUCLEUS

The nucleus is composed of neutrons and protons, which have about the same mass. The atomic number Z of an atom is the number of protons in the nucleus. All atoms of a given element have the same atomic number. For example, plutonium has $Z = 94$.

The atomic weight is the number of protons plus the number of neutrons. Atoms with the same atomic number but with different atomic weights are called "isotopes." For example, plutonium-239 has 94 protons and 145 neutrons and therefore an atomic weight $A = 94 + 145 = 239$. Most elements have more than one stable isotope and several unstable or "radioactive" isotopes.

The unit of nuclear mass is called an "atomic mass unit" or "amu." One amu is defined as exactly one-twelfth of the mass of a carbon-12 atom, which contains six protons and six neutrons.

The unit of energy most commonly used in atomic and nuclear physics is the electron volt (eV). An electron volt is the energy an electron would obtain in falling through an electric potential of 1 volt. Masses can also be expressed in units of energy using the equivalence between mass and energy expressed in the famous formula $E = mc^2$. A proton has a rest mass of 938.272 million electron volts (MeV), while that of a neutron is 939.566 MeV. An electron has a rest mass of only 0.510976 MeV. The atomic mass unit is 931.494 MeV. These units can be transformed into metric units of energy and mass as follows: 1 MeV = 1.602×10^{-13} joules, and 1 MeV = 1.783×10^{-32} kilograms.

The atomic mass unit is less than the average of the masses of the proton and the neutron because it takes energy to break up a nucleus and with that addition of energy comes additional mass.

Using accurately determined masses, one can determine the feasibility of many interactions. For example, one can see that the mass of a neutron is greater than the masses of a proton and an electron combined. Hence, the decay of a neutron into a proton, an electron, and a neutrino (which has zero rest mass) is energetically possible and, in fact, occurs when a neutron is outside a nucleus. However, not all nuclear reactions that would conserve energy occur in nature because other quantities, such as electric charge, must also be conserved.

RADIOACTIVE DECAY

The nuclei of all uranium and plutonium isotopes are unstable in that they each decay into one or more sets of nuclei and other particles, which have a lower total rest mass—the energy difference going into the energy of motion of the resulting nuclei and particles (see figures B-1–4). The isotopes under consideration here undergo three types of radioactive decay: alpha decay, beta decay, and spontaneous fission. Other modes of decay are possible (for example, electron capture and internal transitions), but these will be ignored here.

When an isotope undergoes alpha decay, an alpha particle (helium-4 nucleus) is emitted with an energy of several MeV, the atomic weight of the isotope decreases by four, and the atomic number decreases by two. During beta decay, which is essentially the decay of a neutron inside the nucleus into a proton, a beta particle (electron or positron) and a neutrino* are emitted with a total energy of up to a few MeV; the integer atomic weight remains unchanged (although the actual atomic mass decreases slightly) and the atomic number is increased or decreased by one. The excess of neutrons in the fission products restricts the decays primarily to electrons, and hence the atomic number generally increases. Gamma rays (high-energy photons) are also emitted during alpha and beta decay if the emitted particle does not carry away all of the excess energy, leaving the residual nucleus in an "excited" state that releases the remaining energy in the form of photons. X-rays (intermediate-energy photons) are often emitted if the atomic electrons are disturbed, and bremsstrahlung ("braking radiation"—photons with a continuous energy probability distribution up the full energy of the electron) is emitted as energetic beta particles slow down in matter.

Spontaneous fission is quite different from alpha or beta decay. In this case, the nucleus is so unstable that it splits into two parts. Several gamma rays and neutrons are emitted instantaneously, and the remaining fragments are highly unstable and may undergo beta decay several times. Fission, both spontaneous and neutron- and photon-induced, is described in detail below.

Radioactive decay of all kinds is random in time (the moment of decay for a given atom cannot be predicted), but the rate of decay is proportional to the number of atoms present. This leads to the following relationship: if $N(t)$ is the number of atoms present at time t, and λ is 1/(mean life), the decay rate $Q(t) = N(t) \times \lambda$, and can be expressed by

$$Q(t) = Q(0)\exp(-\lambda t) \tag{B-1}$$

where $Q(0)$is the initial decay rate. If t and λ^{-1} are expressed in seconds or years, the relation gives the number of decays per second or year. $N(t)$ can also be expressed in terms of the mass of the radioactive material present using the fact that 6.02×10^{26} (Avogadro's constant) atoms of an isotope with atomic mass m weighs m kilograms:

* A neutrino, which always accompanies beta decay, is a neutral, almost noninteracting, zero-mass particle.

$$N(t) = K(t)\frac{6.02 \times 10^{26}}{m} \tag{B-2}$$

where $K(t)$ is the mass in kilograms at t, given by

$$K(t) = K(0)\exp(-\lambda t) \tag{B-3}$$

λ can be expressed in terms of the halflife of a material $t_{1/2}$, the time in which half of it decays as

$$\lambda = \frac{\log_e(2)}{t_{1/2}} = \frac{0.693}{t_{1/2}} \tag{B-4}$$

This leads to a value of

$$Q(t) = K(t)\frac{6.02 \times 10^{26}}{m}\frac{0.693}{t_{1/2}} \tag{B-5}$$

Dividing the expression above by the number of seconds in a year, 3.16×10^7, gives the decay rate in number of decays per kilogram per second when the halflife is expressed in years:

$$Q(t) = K(t)\frac{1.32 \times 10^{19}}{t_{1/2}\,m} \tag{B-6}$$

The situation is actually made more complex because many "parent" radioactive isotopes decay to "daughter" (in the terminology of nuclear physics) isotopes that are themselves radioactive, and these decay to other radioactive isotopes, and so on. The decay chains for isotopes of uranium and plutonium contain up to 14 radioactive daughter products. These decay schemes are shown in figures B-1–4. Alpha decays are indicated by a downward arrow; beta (electron) decays by an arrow pointing to the right. Halflives are listed next to each arrow.* When isotopes can decay by either alpha or beta emission, the probability of each type of decay is noted next to the arrows.

* μs = 10^{-6} seconds, ms = 10^{-3} seconds, s = seconds, m = minutes, h = hours, d = days, y = years, ky = 10^3 years, My = 10^6 years, and Gy = 10^9 years.

Spontaneous fission (sf) is indicated by a forked arrow, and the percentage of decays via spontaneous fission is given. Branches of the decay chain that occur with low probability (less than 0.1 percent) are omitted from these diagrams.

The rate of decay of the jth daughter at time t is given by

$$Q_j(t) = Q(0)\prod_{i=1}^{j}\lambda_i\sum_{k=0}^{j}\left[\frac{e^{-\lambda_k t}}{\prod_{\substack{p=0\\p\neq k}}^{j}(\lambda_p - \lambda_k)}\right] \tag{B-7}$$

where λ_i refers to the decay constant of the ith daughter isotope with $i = 0$ for the isotope at the top of the decay chain.[1] The rate of particle emission per kilogram of the parent isotope is given by

$$S_x(t) = \sum_j\left[Q_j(t)\sum_i f_{i,j}\right] \tag{B-8}$$

where S_x is the rate of emission of particle x (photons, alpha particles, beta particles, or neutrons), and $f_{i,j}$ is the probability of the ith emission* of particle type x per decay of the jth radioactive daughter. Values for $f_{i,j}$ can be found in standard references or databases.[2] Our calculations of the photon emissions from WgU and WgPu included over 1,000 emissions from 59 daughter isotopes resulting from decays of various uranium and plutonium isotopes (see figures B-1–4).

The gamma spectrum emitted by WgU is dominated by excited decay products of uranium-238 and uranium-234. The most valuable gamma emission for detection purposes is the 1.001-MeV line of protactinium-234m,† which is a decay product of uranium-238. Higher-energy gamma emissions at 1.399 MeV and 1.832 MeV (from protactinium-234m) or at 1.764 MeV and 2.204 MeV (from bismuth-214, which is a decay product of uranium-234) may also be valuable in situations where a large amount of heavy-metal shielding is present. The decay of uranium-235 results in a negligible emission of high-energy gamma rays; the most prominent emission is at an energy of only 0.186 MeV.

The spectrum of depleted uranium at high energies is dominated by the gamma-ray emissions of protactinium-234m at 1.001, 1.399, and 1.832 MeV.

WgPu lacks intense gamma-ray emissions at energies above 1 MeV. Depending on

* For example, after a particular alpha or beta decay, the residual nucleus may emit gamma rays carrying several different energies.

† Protactinium-234m is a metastable or long-lived excited state of protactinium-234.

the degree of shielding, gamma emissions at 0.414, 0.662, 0.722, or 0.769 MeV will be most valuable for detection purposes. The 0.662- and 0.722-MeV gamma rays are emitted by americium-241, which is a decay product of plutonium-241, and the 0.414- and 0.769-MeV gamma rays are emitted during the decay of plutonium-239.

FISSION

As mentioned above, a heavy nucleus can be so unstable that it fissions, or splits into two nuclei of intermediate weight. There are about 80 different combinations of these intermediate-weight nuclei that can result from the fission of any particular nucleus. Figure B-5 shows the yield of fission fragments as a function of mass for fast-neutron fission of uranium-235 and plutonium-239. Note that the maximum yield for a given mass number is about 7 percent.

Fission occurs spontaneously at a low rate for the heaviest isotopes in the plutonium and uranium decay chains shown in figures B-1–4. Some nuclei, such as the fissile-weapon materials plutonium-239 and uranium-235, can easily be fissioned by incident neutrons of any energy. Others, such as uranium-238, can only be caused to fission with neutrons or photons above a certain energy threshold.

About 200 MeV of energy is released in fission, most of which (about 165 MeV) appears as the kinetic energy of the two fission fragments. The remainder of the energy is associated with prompt and delayed radiations—neutrons, gamma rays, and beta particles. Prompt radiations are those that are emitted during the fission process (actually, within about 10^{-14} seconds of fission), while delayed radiations are those emitted by the unstable fission fragments. Since beta particles are stopped by only a few grams/cm^2 of matter, we concentrate here on neutron and gamma-ray emissions.

Prompt Neutrons

Two to five prompt neutrons are released during fission, depending on the isotope and the energy of the incident particle. Table B-1 gives values for ν, the average number of neutrons emitted per fission, for fissions caused by neutrons of various energies E_n. Also given is the halflife* and ν for spontaneous fission. The rate of neutron production from spontaneous fission of all of the isotopes in a decay chain can be calculated by multiplying $\Sigma f_{i,j}$ by ν_j in equation B-8. Approximate formulas for ν are as a function of the energy E_n of the neutron (in MeV) causing fission are:

$$\text{Uranium-235:} \quad \nu = 2.432 + 0.066E_n \quad (0 \le E_n \le 1) \tag{B-9}$$

$$\text{Plutonium-239:} \quad \nu = 2.874 + 0.138E_n \quad (0 \le E_n) \tag{B-10}$$

The normalized energy spectrum of the neutrons produced by fission is given by

* The halflife for spontaneous fission is that which the nucleus would have if *fission* were its *only* decay mode. Since other types of decay occur with a much higher frequency, the actual halflife is much shorter.

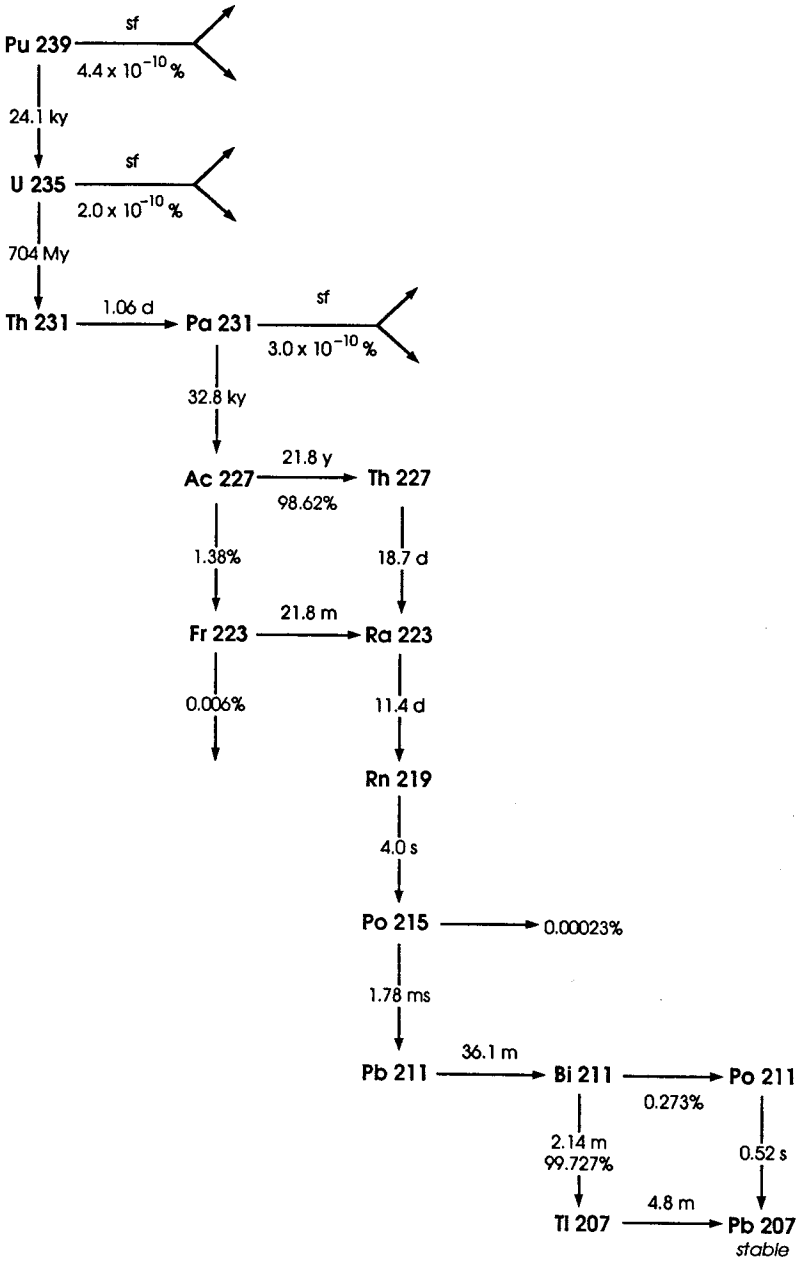

Figure B-1: The decay chain for plutonium-239 and uranium-235

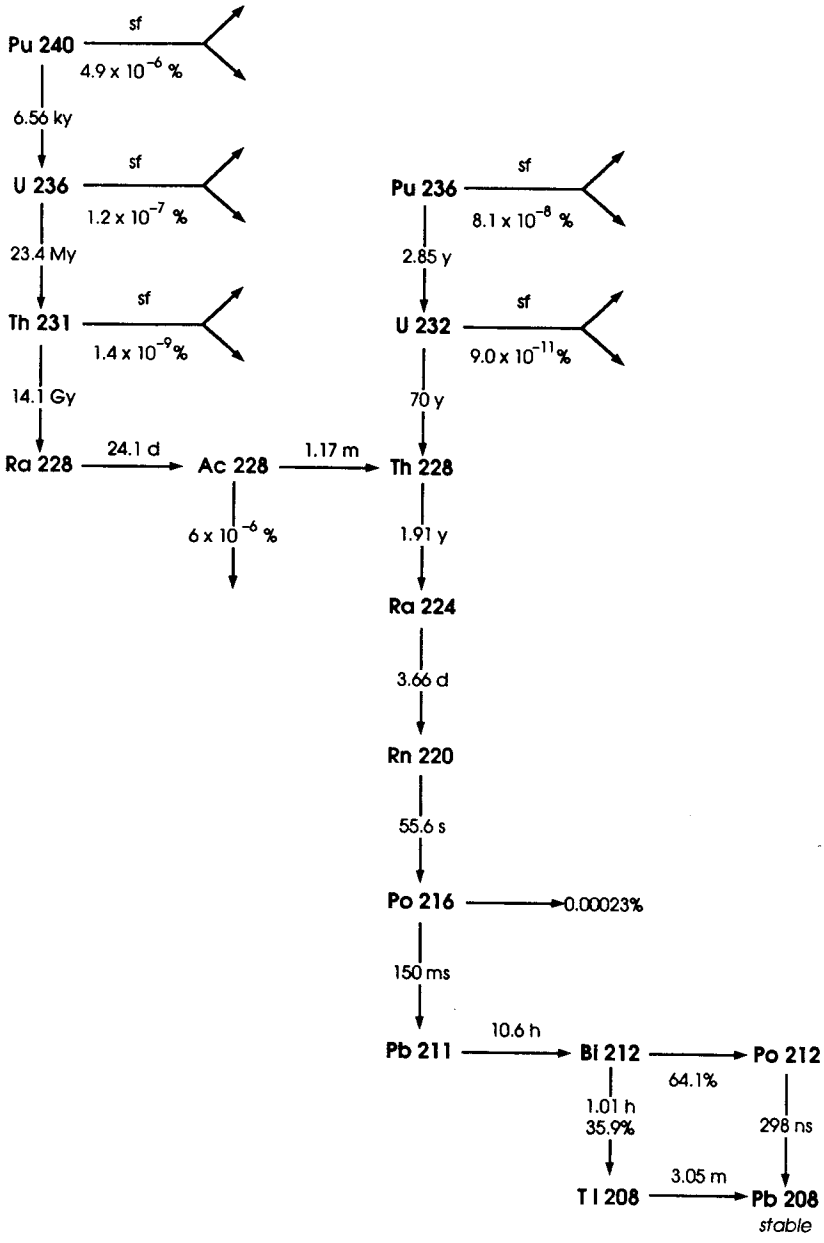

Figure B-2: The decay scheme for plutonium-240

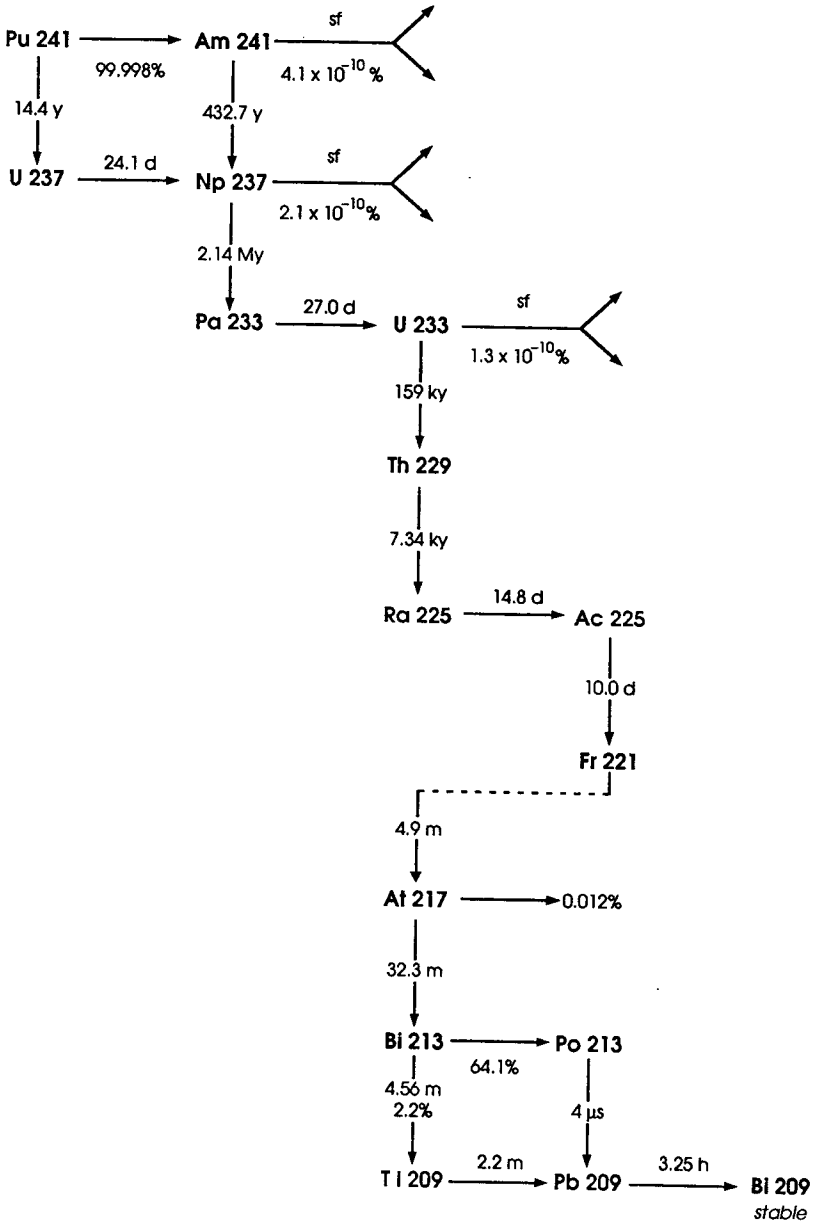

Figure B-3: The decay scheme for plutonium-241

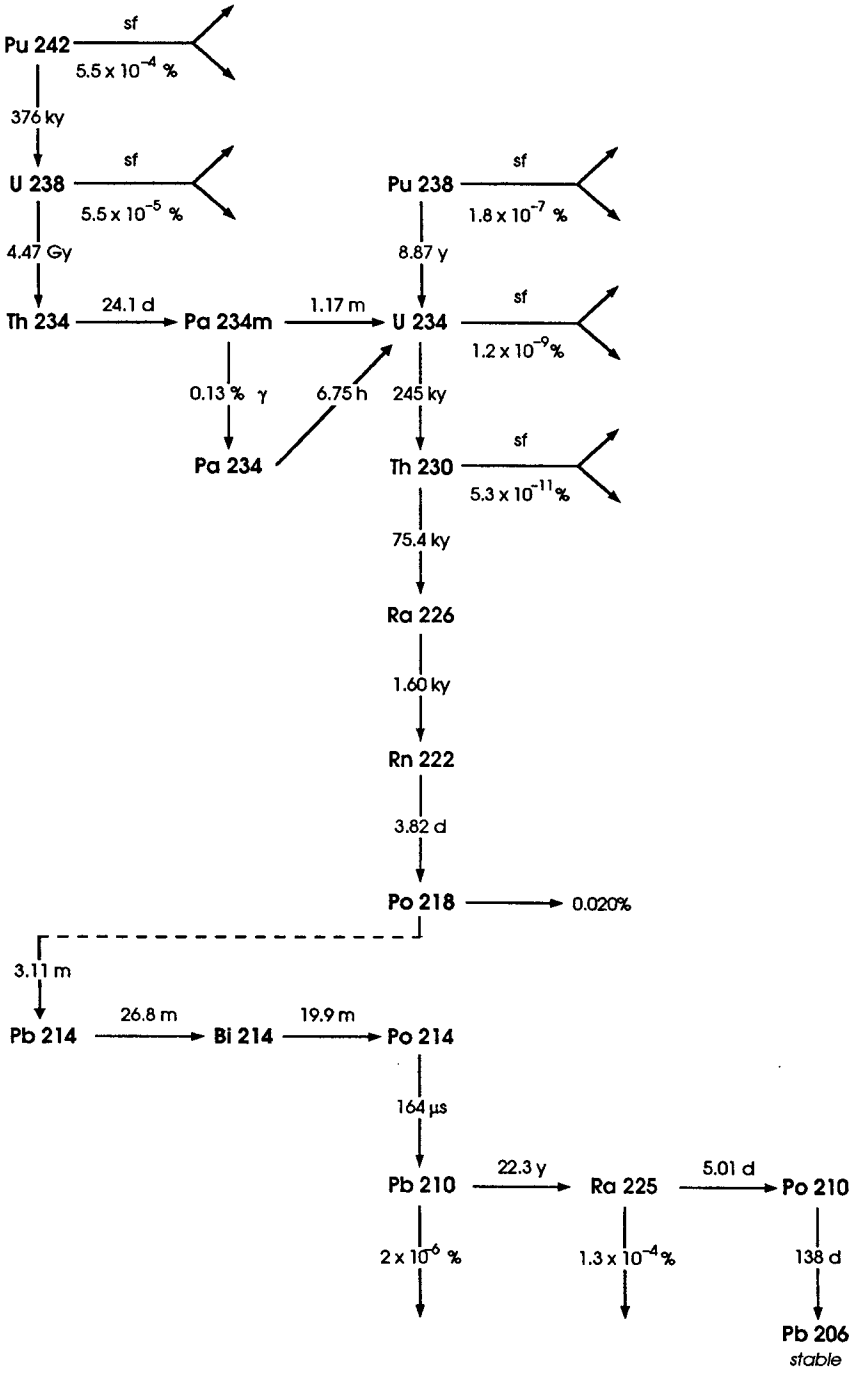

Figure B-4: The decay scheme for uranium-234 and -238 and plutonium-238 and -242

$$\frac{dN(E_n)}{dE_n} = \frac{2}{(\pi\varepsilon^3)^{1/2}} E_n^{1/2} \exp(-E_n/\varepsilon) \qquad \text{(B-11)}$$

where $N(E_n)$ is the number of neutrons with energies between E_n and $(E_n + dE_n)$, and ε, which is equal to two-thirds of the average neutron energy, can be approximated in MeV by[3]

$$\varepsilon = 0.49 + 0.43\sqrt{\nu + 1} \qquad \text{(B-12)}$$

This spectrum peaks at $\sqrt{\varepsilon/2}$ (about 1 MeV), and has a significant tail out to energies of several MeV, which gives a mean neutron energy of about 2 MeV.

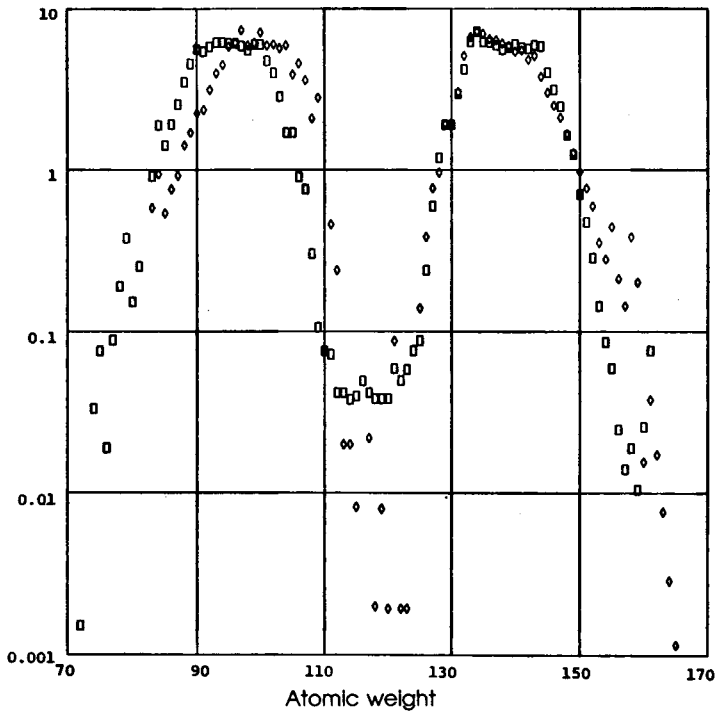

Figure B-5: The percent fission yield by atomic weight of the fission products of uranium-235 (boxes) and plutonium-239 (diamonds) exposed to a fast-reactor neutron spectrum

Prompt Gamma Rays

About seven prompt gamma rays are emitted soon after the prompt neutrons; they carry off the excess energy and angular momentum of the excited fragments after neutron emission is no longer energetically possible. Within the accuracy of the available data, the prompt gamma-ray spectrum can be approximated in (MeV^{-1}) by[4]

$$\frac{dN(E_\gamma)}{dE_\gamma} = 6.7\exp(-1.05E_\gamma) + 30\exp(-3.8E_\gamma) \qquad \text{(B-13)}$$

over the energy range $0.3 < E_\gamma < 7$ MeV. Although this equation is for (n,fission) reactions, it should also apply to spontaneous fission since the spectrum is fairly

Table B-1: The halflife for spontaneous fission (SF) and the average number ν of prompt neutrons released per fission as a function of the energy of the incident neutron

	Halflife for SF *years*	neutrons/fission ν SF	thermal	fission	14 MeV
Uranium-232	$8. \times 10^{13}$	≈2.	-	-	-
Uranium-233	1.22×10^{17}	≈2.	2.49	2.6	4.2
Uranium-234	2.04×10^{16}	≈2.	-	2.4	-
Uranium-235	3.50×10^{17}	1.86	2.42	2.5	4.4
Uranium-236	1.95×10^{16}	1.91	-	2.5	-
Uranium-238	8.20×10^{15}	2.01	-	2.4	-
Neptunium-237	$1. \times 10^{18}$	2.05	2.7	2.8	4.7
Plutonium-236	3.52×10^{9}	2.13	-	-	-
Plutonium-238	4.77×10^{10}	2.22	2.92	-	-
Plutonium-239	5.48×10^{15}	2.16	2.88	3.0	4.8
Plutonium-240	1.31×10^{11}	2.16	2.26	2.9	4.9
Plutonium-241	2.5×10^{15}	2.25	2.93	3.0	5.0
Plutonium-242	6.84×10^{10}	2.15	2.18	-	-
Plutonium-244	6.61×10^{10}	2.30	-	-	-
Americium-241	1.05×10^{14}	2.27	-	-	-

Sources: Edgardo Browne and Richard B. Firestone, *Table of Radioactive Isotopes* (New York: John Wiley & Sons, 1986); Victoria McLane, Charles L. Dunford, and Philip F. Rose, *Neutron Cross Sections, Volume 2: Neutron Cross Section Curves* (Boston, Massachusetts: Academic Press, 1988); R.T. Perry and W.B. Wilson, "Neutron Production from (α,n) Reactions and Spontaneous Fission in ThO_2, UO_2, and $(U,Pu)O_2$ Fuels," LA-8869-MS (Los Alamos, New Mexico: Los Alamos National Laboratory, 1981).

independent of the incident neutron energy. Integration gives 7.2 photons per fission and 7.2 MeV per fission, which compare well with the measured values of 7.4 photons per fission and 7.2 MeV per fission in this energy range.[5]

Delayed Neutrons

The fission fragments mostly relieve themselves of excess neutrons by successive beta decays, converting them into protons through the emission of electrons and neutrinos. However, some isotopes decay to an excited state that can decay by emitting a neutron, which leads to the instantaneous emission of the neutron. Since these neutron emissions immediately follow the beta decay from the parent nucleus, the rate of emission appears to follow the exponential law of radioactive decay for the parent nucleus. The energy spectrum of delayed neutrons is complex because they are emitted by many different isotopes; the average energy is about 0.45 MeV.

The number of delayed neutrons is small: for uranium-235 fissioned by thermal neutrons,* only about 0.016 delayed neutrons are emitted per fission. The ratio of the number of delayed neutrons to the number of prompt neutrons per fission is independent of the energy of the incident neutron. This ratio is 0.0064 for uranium-235, 0.0148 for uranium-238, and 0.0020 for plutonium-239. To predict the rate of delayed neutron emission, ν in the above equations should be multiplied by these ratios.

Delayed Gamma Rays

The unstable fission fragments emit gamma rays as they beta-decay into stable isotopes. Using the observed distribution of some 80 different fission fragments and data on the gamma ray emissions from them and their many radioactive daughters, computer models have predicted precisely the observed delayed gamma-ray spectrum. Delayed gamma ray emission decreases steadily with time at a rate approximately proportional to $t^{-1.2}$, and the energy spectrum of the gamma rays softens with time. This spectrum varies with the isotope undergoing fission and the energy of the incident neutron, since these change the distribution of fission fragments.

For uranium-235, the delayed gamma-ray spectrum from neutron-induced fission can be approximated in MeV^{-1} by[6]

$$\frac{dN(E_\gamma)}{dE_\gamma} = 5.1\exp(-1.03E_\gamma) + 3.8\exp(-1.36E_\gamma) \qquad \text{(B-14)}$$

This equation was fitted to measurements made using sodium iodide detectors, and is

* Thermal neutrons are neutrons that have been slowed down by successive collisions with the nuclei of some material until on average they are as likely to gain as lose energy in subsequent collisions. Their average kinetic energy is then characteristic of the temperature of the material. At a standard temperature of 293 K, they have an average energy of about 0.025 eV.

accurate to within a factor of two over a wide range of energies. Equation B-14 has been normalized to give the measured value of 6.84 MeV per fission when integrated.

The normalized delayed-gamma spectrum is not very different for other isotopes, but the difference in the total energy released can be significant. For example, while the normalized delayed-gamma spectrum from the fission of uranium-238 does not vary by more than 15 percent from that of uranium-235 for gamma-ray energies less than 2 MeV, the total delayed-gamma energy is 60 percent greater. For simplicity, we assume that the delayed-gamma spectrum is given by the above equation multiplied by the ratio of the total energy released relative to uranium-235. The total gamma-ray energy is 10.8 MeV for thorium-232, 4.24 MeV for uranium-233, 10.9 MeV for uranium-238, and 6.15 MeV for plutonium-239.[7]

The total delayed gamma-ray energy from spontaneous fission should be about the same as that from neutron-induced fission of an isotope with one fewer neutron. The total gamma-ray energy from spontaneous fission of plutonium-240 should, for example, be about the same as that from the neutron-induced fission of plutonium-239.

The large number of delayed gamma-ray emissions makes the gamma-ray energy spectrum from fission appear continuous when viewed with a low-resolution detector. A high-resolution instrument would reveal the line spectra from the radioactive decay of various fission products.

The most detectable delayed-gamma emission is probably the 1.597-MeV emission from lanthanum-140, which is emitted in 5–6 percent of all fissions (fairly independently of the isotope undergoing fission).[8] We have used this line to estimate the detectability of delayed gamma rays with high-resolution detectors.

INTERACTION OF RADIATION WITH MATTER

Neutron Interactions

As neutrons pass through matter, they can interact in two distinct ways. Often the interaction is an "elastic" scattering in which the neutron bounces off a nucleus without giving the target nucleus any internal "excitation" energy—as if it were a small billiard ball colliding with a large one. One can use this billiard-ball model to calculate the energy that a neutron would lose in an elastic collision. Using a nonrelativistic approximation, which is valid at the neutron energies with which we are dealing, we obtain the relationship:

$$p_2 = \frac{p_1}{M + m}\left[m\sin\theta + (M^2 - m^2\cos^2\theta)^{1/2}\right] \qquad \text{(B-15)}$$

where p_1 is the initial momentum of a particle of mass m, and p_2 is its final momentum after it has scattered elastically at an angle θ from a particle of mass M. The result for the case where the scattered particle bounces back in the direction from which it came ($\theta = 180°$) illustrates the sensitivity of the energy loss to the relative mass of the first particle with which it collides:

$$p_2 \underset{\theta=180^\circ}{=} p_1\left[\frac{M-m}{M+m}\right]^{1/2} \tag{B-16}$$

If $M = m$ the final momentum is zero; if $M >> m$, $p_2 \approx p_1$. This is why light elements are better at slowing down neutrons.

The neutron can also bounce off a nucleus inelastically and transfer energy to the nucleus's interior. This nuclear excitation will result in the later emission of one or more gamma rays by the nucleus.

Finally, the neutron can be absorbed by the nucleus, raising its atomic weight by one unit. As a result of the addition of the absorbed neutron, the new nucleus may emit a gamma ray, an alpha particle, a neutron, or it may fission. A neutron's ability to cause these results depends critically on its kinetic energy at the time it is absorbed. These prompt interactions are described by the notation "(n,γ)," "(n,α)," "(n,n)," "$(n$,fission)," and so on. Neutron absorption may also create an unstable nucleus that decays with a characteristic halflife by the emission of an electron (beta decay).

A term frequently used to describe the likelihood of various nuclear interactions is "cross section." The probability that an interaction will take place is expressed as proportional to an equivalent cross-sectional area centered on the target nucleus. An effective cross section can represent the probability for elastic scattering, (n,α) interactions, and so on. It is as if the total cross-sectional area for a collision were subdivided into areas for all of the possible interactions. The cross sections change as a function of energy.

The radius of a proton or a neutron is slightly larger than 10^{-13} centimeters, which corresponds to geometric cross sections of a few times 10^{-26} square centimeters. Because nuclear densities do not vary much (except for the very lightest nuclei), the radii of nuclei generally increase as the cube root of their masses. The unit used to measure nuclear cross sections is the "barn," which is equal to 10^{-24} square centimeters. (Atomic radii are much larger than nuclear radii—about 10^{-8} centimeters—with corresponding geometric cross sections of a few times 10^{-16} square centimeters.)

Of particular interest for our discussion are the total cross section for interaction and the elastic scattering cross section. If we wish to know the mean-free-path distance l a neutron can proceed through a material without interacting, we can use the measured total cross section σ and the density of nuclei in a material N to calculate $l = 1/N\sigma$. Then the number of neutrons $n(d)$ in a beam originally containing $n(0)$ neutrons decreases with distance d into the material as

$$n(d) = n(0)\exp(-d/l) \tag{B-17}$$

The quantities d and l can be measured either in centimeters or in grams/cm^2.

Thus, for example, in a material with a density of 10 grams/cm^2, a mean free path of 1 centimeter would equal a mean free path of 10 grams/cm^2. The unit of grams/cm^2 is used in calculations of shielding masses.

Neutrons are particularly useful for fissile-material detection because their range in matter is quite large. Table B-2 gives the mean free paths of neutrons of different energies in various materials, both in centimeters and in grams/cm^2. This table gives the interaction length for undergoing any kind of interaction, including elastic scattering, absorption, or fission. If the neutron undergoes elastic scattering, there will still be a neutron moving through the material in a different direction and at a lower energy. The mean free paths for absorption of the neutron (that is, until it disappears) are therefore generally much longer than those given in the tables.

The fact that neutrons can interact in so many different ways, and that the probability of these many interactions is a complicated function of the neutron energy, makes it almost impossible to estimate the emission of neutrons from the weapon models by hand. We have used the computer program TART to trace the tortuous paths of neutrons emitted by fissile material and to estimate the number of neutrons escaping from the weapon models.[9]

Gamma-rays

Gamma rays (high-energy photons) are essentially pure energy. A gamma ray can transfer all of its energy to an atomic electron, ejecting it from its bound state as part of the atom. This is called "photoelectric absorption" and is a major cause of energy loss of low-energy gamma rays (below 1 MeV). Since photoelectric effect is most probable when the binding energy of the electrons is comparable to the energy of the absorbed photon and high-Z atoms contain electrons that are more tightly bound, photoelectric absorption occurs much more readily at high energies in high-Z atoms.

If the gamma-ray energy is not totally absorbed by the ejection of the electron, the interaction is not an absorption but a "Compton" scattering, with the gamma ray proceeding in a new direction and at a lower energy after the interaction. For scattering from an electron that is unbound or whose binding energy may be neglected, the energy E of the scattered gamma ray is given by

$$E = \frac{E_0}{1 + (E_0/mc^2)(1 - \cos\theta)} \qquad \text{(B-18)}$$

where E_0 is the original energy of the gamma ray, m is the electron mass, c the velocity of light, and θ is the scattering angle. Compton scattering is dominant at energies of about 1 MeV in light elements.

At energies greater than 1 MeV, pair-production starts playing a role in absorption and, at 100 MeV, it is the dominant method of absorption. In this interaction the gamma ray ceases to exist—it is transformed into an electron and an anti-electron (positron). Below 1.02 MeV, a gamma ray does not possess enough energy to produce

Table B-2: Mean free path of neutrons for various elements and isotopes

a: *grams/cm²*

	Neutron energy *eV*					
	10^{-1}	10	10^3	10^5	10^6	10^7
Hydrogen	0.042	0.042	0.042	0.083	0.42	2.4
Helium	8.9	8.9	8.9	8.3	1.0	4.2
Lithium	0.022	0.20	1.7	6.7	8.3	5.6
Beryllium	2.5	2.5	2.5	2.8	4.5	10.
Boron-10	0.0056	0.067	0.83	3.3	7.9	14.
Boron-11	3.7	3.7	3.7	4.2	8.3	15.
Carbon	4.7	4.7	4.8	4.8	7.7	17.
Nitrogen	1.9	2.1	2.6	4.7	12.	15.
Oxygen	6.7	6.7	6.7	6.7	3.8	16.
Aluminum	28.	30.	30.	11.	15.	22.
Iron	7.8	9.3	12.	23.	31.	40.
Copper	13.	15.	14.	21.	29.	42.
Tungsten	31.	61.	22.	26.	44.	61.
Lead	31.	31.	31.	31.	72.	58.
Uranium-235	0.78	9.8	13.	36.	56.	65.
Uranium-238	40.	26.	16.	26.	44.	57.
Plutonium-239	0.44	1.3	13.	27.	57.	57.

b: *centimeters*

	Density	Neutron energy *eV*					
	g/cm³	10^{-1}	10	10^3	10^5	10^6	10^7
Hydrogen *liquid*	0.071	0.59	0.59	0.59	1.2	5.9	34.
Helium *liquid*	0.125	71.	71.	71.	66.	8.0	34.
Lithium	0.534	0.42	0.38	3.2	13.	16.	11.
Beryllium	1.85	1.4	1.4	1.4	1.6	2.4	5.6
Boron-10	2.17	0.0024	0.028	0.35	1.4	3.4	5.9
Boron-11	2.38	1.6	1.6	1.6	1.8	3.6	6.5
Carbon	2.27	2.1	2.1	2.1	2.1	3.4	7.4
Nitrogen *liquid*	0.81	2.4	2.6	3.2	5.8	15.	18.
Oxygen *liquid*	1.14	5.9	5.9	5.9	5.9	3.3	14.
Aluminum	2.7	10.	11.	11.	4.2	5.6	8.3
Iron	7.9	0.99	1.2	1.5	3.0	3.9	5.1
Copper	9.0	1.5	1.7	0.47	2.4	3.2	4.7
Tungsten	19.3	1.6	3.2	1.1	1.3	2.3	3.2
Lead	11.4	2.8	2.8	2.8	2.8	6.4	5.0
Uranium-235	18.71	0.041	0.52	0.69	1.9	3.0	3.4
Uranium-238	18.95	2.1	1.4	0.84	1.4	2.3	3.0
Plutonium-239	19.84	0.023	0.07	0.70	1.4	3.0	3.0

Source: Hildebrand and Leith, *Physical Review* **80**, 842 (1950).

a positron/electron pair (the rest mass of an electron or positron is 0.51 MeV). Pair production is greatly enhanced in the strong electric field of a high-Z nucleus. At higher energies, pair production can lead to electromagnetic showers, because the resulting high-energy electrons and positrons generate high-energy gamma rays (which can cause additional pair production) as they interact with the surrounding medium.

The combined cross section of all of these interactions is called the total cross section: table B-3 lists the mean free paths of gamma rays in various materials.

High-resolution gamma-ray detectors are most effective when they are used to detect emissions at particular energies rather than the emissions at all energies. For

Table B-3: Mean interaction free path of gamma rays for various elements

a: *grams/cm²*

	Photon energy *MeV*								
	0.1	0.2	0.4	0.6	1.0	2.0	4.0	6.0	10.0
Hydrogen *liquid*	3.4	4.1	5.3	6.3	7.9	11	17	22	31
Beryllium	7.6	9.2	12	14	18	25	38	47	62
Carbon	6.7	8.2	11	12	16	23	33	41	52
Aluminum	6.2	8.3	11	13	16	23	32	38	44
Iron	2.9	7.2	11	13	17	24	30	33	34
Copper	2.3	6.8	12	13	17	24	30	32	33
Tungsten	0.24	1.4	5.7	9.9	16	23	25	24	22
Lead	0.19	1.1	4.8	8.8	15	22	24	23	20
Uranium	0.94	0.85	3.9	7.4	13	21	23	22	20

b: *centimeters*

	Photon energy *MeV*								
	0.1	0.2	0.4	0.6	1.0	2.0	4.0	6.0	10.0
Hydrogen	48	58	74	88	110	160	240	317	440
Beryllium	4.1	5.0	6.4	7.0	9.6	14	20	26	34
Carbon	3.0	3.6	4.7	5.5	7.0	10	15	18	23
Aluminum	2.3	3.1	4.0	4.8	6.0	8.6	12	14	16
Iron	0.37	0.92	1.4	1.7	2.1	3.0	3.9	4.2	4.3
Copper	0.26	0.76	1.2	1.5	1.9	2.7	3.4	3.6	3.7
Tungsten	0.012	0.073	0.30	0.51	0.81	1.2	1.3	1.2	1.1
Lead	0.017	0.098	0.42	0.77	1.3	1.9	2.1	2.0	1.8
Uranium	0.050	0.046	0.21	0.39	0.71	1.1	1.2	1.2	1.0

Source: J.H. Hubbell, *International Journal of Applied Radiation and Isotopes*, **33**, 1269 (1982).

this reason, it is relatively simple to use the mean free paths given in table B-3 to estimate the gamma-ray emissions from the weapon models, because only those gamma rays that have not interacted at all with the weapon materials will reach the detector undegraded in energy, and can be counted in the "full-energy peak."

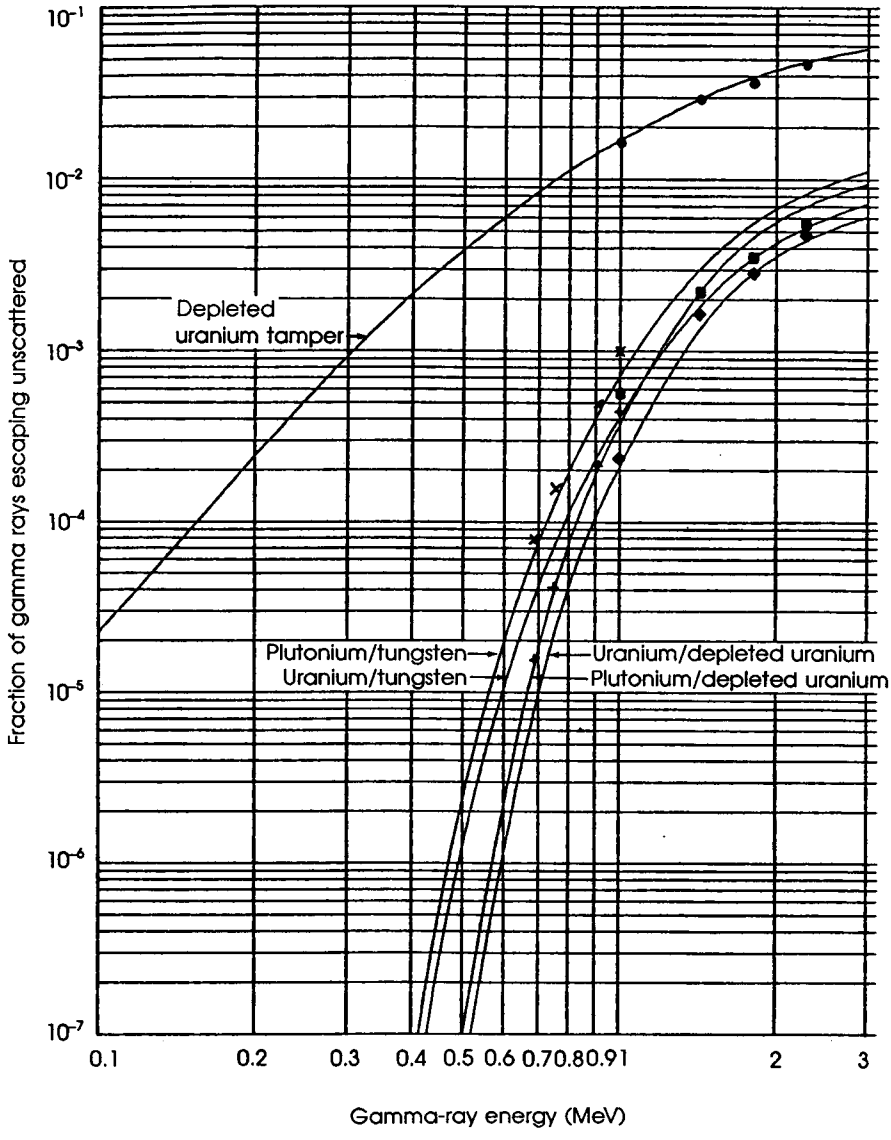

Figure B-6: The fraction of gamma rays produced in the fissile material or depleted uranium that escape unscattered from each weapon model as a function of gamma-ray energy

The fraction of gamma rays escaping from the weapon can be given as

$$f = G\prod_k F_k \tag{B-19}$$

where G is the self-shielding factor (that is, the fraction of gamma rays that escape from the fissile material unscattered) and F_k is the fraction of gamma rays that pass through the kth material between the source and the detector unscattered.

For a spherical shell source of inside radius r and outside radius R inside a concentric spherical shell absorber with inside radius a and outside radius b, it can be shown that F is equal to[10]

$$\frac{3}{R^3 - r^3}\int_r^R\int_0^{\pi/2} x^2\sin\theta \exp(\mu[a^2 - x^2\sin^2\theta]^{1/2} - \mu[b^2 - x^2\sin^2\theta]^{1/2})\,d\theta\,dx \tag{B-20}$$

where μ is the linear attenuation coefficient of the absorber (1/mean free path). If the source is thin ($[R - r] << r$), then this can be approximated by

$$\int_0^{\pi/2}\sin\theta \exp(\mu[a^2 - \rho^2\sin^2\theta]^{1/2} - \mu[b^2 - \rho^2\sin^2\theta]^{1/2})\,d\theta \tag{B-21}$$

where ρ is the average radius $(R + r)/2$. If the radius of the source is small compared with that of the absorber ($b >> R$) then

$$F \approx \exp(-\mu[b - a]) \tag{B-22}$$

Similarly, the self-shielding factor G is given by

$$\frac{3}{2(R^3 - r^3)}\left[\int_r^R\int_0^{b\cos(r/x)+\pi/2} x^2\sin^2\theta \exp(\mu\, x\cos\theta - \mu[R^2 - x^2\sin^2\theta]^{1/2})\,d\theta\,dx + \int_r^R\int_{b\cos(r/x)+\pi/2}^{\pi} x^2\sin\theta \exp(\mu\, x\cos\theta - \mu[R^2 - x^2\sin^2\theta]^{1/2} + 2\mu[r^2 - x^2\sin^2\theta]^{1/2})\,d\theta\,dx\right] \tag{B-23}$$

Equation B-23 can be approximated by

$$G \approx \frac{1}{\beta\mu\delta r}(1 - \exp[-\beta\mu\delta r]) \qquad \text{(B-24)}$$

where $\delta r = (R - r)$ and β ranges between $^4/_3$ and 4, depending on the aspect ratio of the source. For spheres, $\beta = {}^4/_3$; for thin shells, $\beta = 4$. An approximate expression for β is

$$4 - \frac{8}{3}\exp\left(-0.57\sqrt{r/\delta r}\right) \qquad \text{(B-25)}$$

Equations B-24 and B-25 are good to within ±10 percent over a wide range of $\mu\delta r$ ($\mu\delta r > 0.01$) and $r/\delta r$ ($r/\delta r < 1{,}000$).

Figure B-6 gives the fraction of gamma rays escaping from the depleted uranium tamper and from the fissile cores of each weapon model as a function of the gamma-ray energy as predicted by equations B-19, B-20, and B-23. Also shown are the results of TART calculations for several energies. Note that there is good agreement between the analytical expressions and the Monte Carlo results of TART, with TART giving estimates 10–30 percent higher.

NOTES AND REFERENCES

1. This relationship was first derived by Bateman in 1910; its derivation is well described in Evans, *The Atomic Nucleus* (New York: McGraw-Hill, 1955).

2. The values used in this study are taken from Edgardo Browne and Richard B. Firestone, *Table of Radioactive Isotopes* (New York: John Wiley & Sons, 1986); C. Michael Lederer and Virginia Shirley, *Table of Isotopes*, 7th edition (New York: John Wiley & Sons, 1978); and from an online data library maintained by the National Nuclear Data Center at Brookhaven National Laboratory.

3. G. Robert Keepin, *Physics of Nuclear Kinetics* (Reading, Massachusetts: Addison-Wesley, 1965), p.46.

4. Ibid., p.67.

5. Ibid, p.66.

6. Fit by the authors to data in figure 5-1 of Keepin, *Physics of Nuclear Kinetics*. Data for five decay times ranging from 1.7 to 1,000 seconds were integrated numerically over time for each of 17 energy groups ranging from 0.11 to 6.5 MeV.

7. Keepin, *Physics of Nuclear Kinetics*, p.136.

8. This photon is emitted in 5.74, 5.14, and 5.50 percent of uranium-235, plutonium-239, and uranium-238 fast fissions respectively. In some circumstances, other photon emissions may be somewhat more detectable. For example, the 3.317-MeV photon emitted from rubidium-90 in 0.83 percent of uranium-235 and 0.34 percent of plutonium-239 fast-fissions would be about as detectable after passing through 10 centimeters of uranium.

9. Ernest F. Plechaty and John R. Kimlinger, "TARTNP: A Coupled Neutron-Photon Monte Carlo Transport Code," UCRL-50400, volume 14, (Livermore, California: Lawrence Livermore National Laboratory, 4 July 1976).

10. This equation is valid for detectors located at a distance from the source and shielding materials that is large compared to their dimensions. Although equations B.20 and B.23 must have been derived before, we could find no solution to this problem in the standard references on radiation shielding.